# Assignment Problem

C.Mohanraja, M Sc., MSc(M).,M Tech., MBA., MPhil.,
Assistant Professor in Computer Science,
St. Joseph's College (Autonomous),
Tiruchirappallai-620 002.

# Hungarian Method

# Introduction

- The assignment problem is a special type of transportation problem
- The objective is to minimize the cost or time of completing a number of jobs by a number of persons.
- The assignment model is useful in solving problems such as, assignment of machines to jobs, assignment of salesmen to sales territories, travelling salesman problem, etc.

# Algorithm

Step1:

Identify the minimum element in each row and subtract it from every element of that row.

Step2:

Identify the minimum element in each column and subtract it from every element of that column.

Step3:

Make the assignments for the reduced matrix obtained from **steps 1 and 2** in the following way:

- For each row or column with a single zero value cell that has not be assigned or eliminated, box that zero value as an assigned cell.

# Cont……

- For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
- If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, then you are at liberty to choose the cell arbitrarily for assignment.
- The above process may be continued until every zero cell is either assigned or crossed (X).

Step 4:

An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.

# Cont……

Step5:

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:

- Mark all the rows that do not have assignments.
- Mark all the columns (not already marked) which have zeros in the marked rows.
- Mark all the rows (not already marked) that have assignments in marked columns.
- Repeat steps 5 (i) to (iii) until no more rows or columns can be marked.
- Draw straight lines through all unmarked rows and marked columns.

# Cont……

Step6:

Select the smallest element from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

Step7:

Go to **step 3** and repeat the procedure until you arrive at an optimal assignment.

# Cont……

1)Solve the following Assignment Problem for Minimization

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 20 | 25 | 22 | 28 |
| B | 15 | 18 | 23 | 17 |
| C | 19 | 17 | 21 | 24 |
| D | 25 | 23 | 24 | 24 |

# Cont……

## Solution:

### Step 1:

- Identify the minimum element in each row and **subtract** it from every element of that row. The result is shown in the following table.

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 5 | 2 | 8 |
| B | 0 | 3 | 8 | 2 |
| C | 2 | 0 | 4 | 7 |
| D | 2 | 0 | 1 | 1 |

# Cont……

## Step 2:

Identify the minimum element in each column and subtract it from every element of that column.

| | Job | | | |
|---|---|---|---|---|
| **Person** | **1** | **2** | **3** | **4** |
| **A** | 0 | 5 | 1 | 7 |
| **B** | 0 | 3 | 7 | 1 |
| **C** | 2 | 0 | 3 | 6 |
| **D** | 2 | 0 | 0 | 0 |

# Cont……

## Step 3:

Make the assignments for the reduced matrix obtained from **steps 1 and 2** in the following way:

- For each row or column with a single zero value cell that has not be assigned or eliminated, box that zero value as an assigned cell.
- For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
- If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, choose the cell arbitrarily for assignment.
- The above process may be continued until every zero cell is either assigned or crossed (X).

# Cont……

## Step 4:

- An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | | 5 | 1 | 7 |
| B | | 3 | 7 | 1 |
| C | 2 | | 3 | 6 |
| D | 2 | | | |

# Cont……

## Step 5:

- Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:
  - Mark all the rows that do not have assignments.
  - Mark all the columns (not already marked) which have zeros in the marked rows.
  - Mark all the rows (not already marked) that have assignments in marked columns.
  - Repeat steps 5 (ii) and (iii) until no more rows or columns can be marked.
  - Draw straight lines through all unmarked rows and marked columns.

# Cont……

| Job | | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 5 | 1 | 7 |
| B | 0 | 3 | 7 | 1 |
| C | 2 | 0 | 3 | 6 |
| D | 2 | 0 | 0 | 0 |

# Cont……

## Step 6:

- Select the smallest element (i.e., 1) from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | 0 | 4 | 0 | 6 |
| B | 0 | 2 | 6 | 0 |
| C | 3 | 0 | 3 | 6 |
| D | 3 | 0 | 0 | 0 |

# Cont……

- Now again make the assignments for the reduced matrix.

**Final Table**

| | Job | | | |
|---|---|---|---|---|
| Person | 1 | 2 | 3 | 4 |
| A | | 4 | | 6 |
| B | | 2 | 6 | |
| C | 3 | | 3 | 6 |
| D | 3 | | | |

# Cont……

- Since the number of assignments is equal to the number of rows (& columns), this is the optimal solution.
- The total cost of assignment = A1 + B4 + C2 + D3
- Substituting values from original table: 20 + 17 + 17 + 24 = Rs. 78.

# Thank You